Phyllis J. Cassidy
City College of CUNY
May 14, 2005

A converse of the Ritt-Raudenbush Basis Theorem for commutative differential Hopf algebras.

Abstract: The formulation in [1] of a Galois theory of parametrized linear differential equations requires a more Tannakian approach to the theory of linear differential algebraic groups. This re-casting of the theory of linear differential algebraic groups is joint work with Jerald J. Kovacic. In particular, the proof of the fundamental theorem of Galois theory requires an affirmative answer to the following question:

Let G be a linear differential algebraic group, and, H a normal differential algebraic subgroups. Is the ring of differential invariants of H, acting on the coordinate algebra of G by the regular representation, differentially finitely generated?

Let F be a differential field of characteristic zero, and, let ∂ be the finite family of commuting derivation operators on F defining its differential structure. (We will indicate differential structures by the prefix ∂-.) A Hopf $\mathcal{F}$-algebra R is a ∂-Hopf $\mathcal{F}$-algebra if the comultiplication, counit, and antipode are ∂-homomorphisms, *i.e.*, they commute with the derivations in ∂. We shall assume that R is commutative. An ideal I is a ∂-ideal if it is stable under the derivations in ∂. It is ∂-finitely generated if there is a finite family of elements of I such that the ideal I is generated by the members of the family and all their ∂-derivatives. (I need not be a finitely generated ideal.) Since the characteristic of R is 0, the radical of a ∂-finitely generated ∂-ideal I of R is also a ∂-ideal and is said to be ∂-finitely generated. In a similar vein, the ∂-F-algebra R is ∂-finitely generated over F if R is generated as an F-algebra by a *finite* family and all the higher derivatives of this family. (It may be *infinitely* generated as an F-algebra.) The *Ritt-Raudenbush Basis Theorem*, proved in the 1930's, which made differential algebraic geometry possible says that every ∂-finitely generated commutative ∂-F-algebra is ∂-Noetherian.

In this talk, we present the main ideas of the proof of a theorem that will both answer affirmatively the above question, and, is a converse of the Ritt-Raudenbush basis theorem for commutative ∂-Hopf F-algebras: Every ∂-Hopf F-subalgebra of a ∂-Noetherian ∂-Hopf F-algebra is ∂-finitely generated

as a ∂-F-algebra. The proof rests easily on three theorems: The Ritt-Raudenbush basis theorem [3, 4], a theorem of Cartier [2], and a theorem of Takeuchi [5].

[1] Cassidy, P. J., and, Michael F. Singer, Galois theory of parametrized differential equations and linear differential algebraic groups, submitted to the *Proceedings* of the Conference, *Singularités des équations différentielles, systèmes intégrables et groupes quantiques*, November 24-27, Strasbourg, Fr.

[2] Oort, F., Algebraic group schemes in characteristic zero are reduced, *Inventiones math., 2 (1966)*, 79-80.

[3] Raudenbush, H. W., On the analog for differential equations of the Hilbert-Netto theorem, *Bull. Amer. Math. Soc.*, 42 (1936), 371-373.

[4] Ritt, J. F., *Differential Equations from the Algebraic Standpoint*, Amer. Math. Soc. Colloq. Pub. 14, Amer. Math. Soc., New York, 1934.

[5] Takeuchi, M., A correspondence between bi-ideals and sub-Hopf algebras, *Manuscripta Math. 7 (1972), 251-270.*

[6] Sweedler, Moss E., *Hopf Algebras*, W. A. Benjamin, Inc., 1969.