SECTIONS OF A DIFFERENTIAL SPECTRUM

ALEXEY OVCHINNIKOV

We construct sections of a structure presheaf $\mathcal{O}$ on a differential spectrum using only localization and projective limits. For this purpose we introduce a special form of a multiplicative system generated by one differential polynomial and call it *D-localization*. So, first we define "functions" on principal open subsets using this localization and then, as in algebraic geometry, take the projective limit. This technique allows us to construct sections of a differential spectrum of a differential ring $\mathcal{R}$ without computation of $\operatorname{diffspec}\mathcal{R}$.

We compare our construction with Kovacic's structure sheaf $\mathcal{O}_X$ and discuss various computational aspects. Our method of constructing sections does not give a sheaf of functions for all differential rings. Nevertheless, we prove the following theorems.

Theorem 1. *Let $\mathcal{R}$ be a factorial differential ring. Then the structure presheaf $\mathcal{O}$ is a sheaf of differential rings on* $\operatorname{diffspec}\mathcal{R}$*.*

Theorem 2. *Let $\mathcal{R}$ be a factorial differential ring. Then structure sheaf $\mathcal{O}$ on* $\operatorname{diffspec}\mathcal{R}$ *coincides with Kovacic's structure sheaf $\mathcal{O}_X$ on $X = \operatorname{diffspec}\mathcal{R}$.*

We also discuss the following example (counter-example for Theorem 1) appeared before in [4, 6]. We give a *reasoning* for its appearing.

Example 1. Let $\mathfrak{a} = \{yy'' - y'(y'+1)\} \subset k\{y\}$ and $\mathcal{R} = k\{y\}/\{yy'' - y'(y'+1)\}$.

In the case when our construction that actually coincides with one, obtained in [2], does not give a sheaf one can apply sheafication and obtain exactly the same as Kovacic's structure sheaf $\mathcal{O}_X$.

References

[1] Hartshorne R., *Algebraic Geometry,* Springer-Verlag, New York, 1977.

[2] Keigher W.F., *On the structure presheaf of a differential ring,* J. Pure Appl. Algebra 27, 1983, 163-172.

[3] Kolchin E.R., *Differential Algebra and Algebraic Groups,* Academic Press, 1973.

[4] Kolchin E.R., *Differential Algebraic Groups,* Pure Appl. Math., vol. 114, Academic Press, Orlando, FL, 1985.

[5] Kovacic J.J., *The differential Galois theory of strongly normal extensions,* Trans. AMS Vol 355, Number 11, 2003, 4475-4522.

[6] Kovacic J.J., *Global sections of diffspec,* J. Pure and Applied Alg., 171, 2002, 265-288.

[7] Kovacic J.J., *Differential Schemes,* Differential Algebra and Related Topics, Proceedings of the International Workshop, Rutgers University, Newark, November 2-3, 2002.

[8] Ritt J.F., *Differential Algebra,* volume XXXIII of Colloquium Publications. New York, American Mathematical Society, 1950.

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[9] Shafarevich I.R., *Basic Algebraic Geometry: Schemes and Complex Manifolds, Vol. 2*, Springer-Verlag New York, 1996.

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